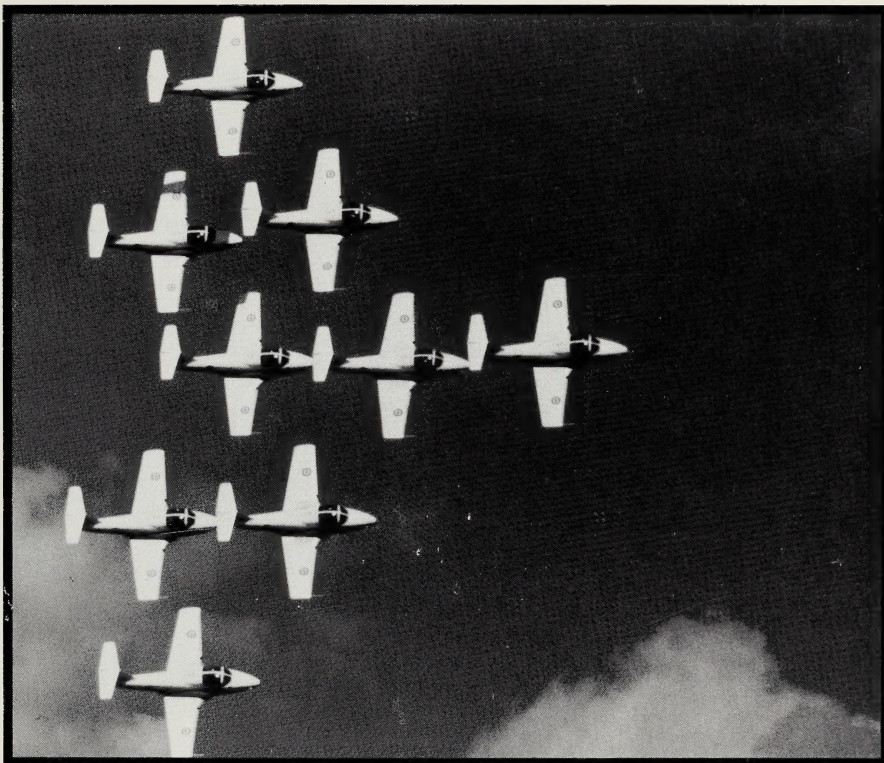




Modules 1-7



Learning Facilitator's Manual



Distance
Learning

Alberta
EDUCATION

Mathematics 9

LEARNING FACILITATOR'S MANUAL



Note

This Mathematics 9 Learning Facilitator's Manual contains answers to teacher-assessed assignments and the final test; therefore, it should be kept secure by the teacher. Students should not have access to these assignments or the final test until they are assigned in a supervised situation. The answers should be stored securely by the teacher at all times.

IT IS STRICTLY PROHIBITED TO COPY ANY PART OF THESE MATERIALS UNDER THE TERMS OF A LICENCE FROM A COLLECTIVE OR A LICENSING BODY.

This document is intended for	
Students	
Teachers (Mathematics 9)	✓
Administrators	
Parents	
General Public	
Other	

Mathematics 9
Learning Facilitator's Manual
Modules 1-7
Alberta Distance Learning Centre
ISBN No. 0-7741-0846-0

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Teachers

Register with the Alberta Distance Learning Centre

The Alberta Distance Learning Centre is dedicated to upgrading and continually improving your Learning Facilitator's Manual so that it accurately reflects any necessary revisions we have had to make in the student module booklets, assignment booklets, or the sample final test. The types of revisions that will be made are those that make the course more accurate, current, or more effective.

The ADLC will send you the **latest enhancements or minor upgrades** for your Learning Facilitator's Manual if you return the following registration card to: Alberta Distance Learning Centre, Box 4000, Barrhead, Alberta, T0G 2P0, Attention: Instructional Design and Development.

✂

ADLC Learning Facilitator's Manual Registration Card	
_____ First Name	_____ Surname
_____ School Name	_____ School Phone Number
_____ School Address	
_____ City	_____ Postal Code
_____ Course Title	_____ Approximate Date of Purchase

✂



You can help ensure that distance learning courseware is of top quality by letting us know of areas that need to be adjusted. Call the Alberta Distance Learning Centre free of charge by using the RITE line and ask for the Editing Unit. Also, a teacher questionnaire has been included at the back of most Learning Facilitator's Manuals. Please take a moment to fill this out.



We look forward to hearing from you!



Register with the

Alcohol Distance Learning Course

YOU WILL LEARN ABOUT:

The Alcohol Distance Learning Course is a self-paced, self-administered course that provides a comprehensive overview of the legal and ethical aspects of alcohol and drug abuse. The course is designed for healthcare professionals who are required to complete a certain number of continuing education hours. The course is available in both English and Spanish.

The course is available in both English and Spanish. The course is designed for healthcare professionals who are required to complete a certain number of continuing education hours. The course is available in both English and Spanish.

Alcohol Distance Learning Course	
Participant Information	
Name	
Address	
City	
State	
Zip	
Phone	
Fax	
E-mail	
Signature	
Date	
Signature of Instructor	
Date	

The course is available in both English and Spanish. The course is designed for healthcare professionals who are required to complete a certain number of continuing education hours. The course is available in both English and Spanish.

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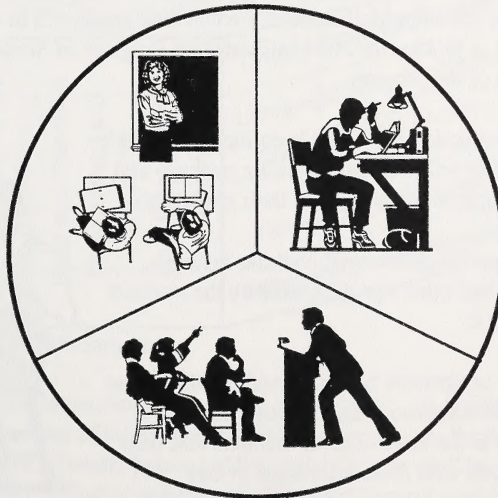
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Introduction

A survey of these course materials will confirm that this new learning package has been specially designed for many kinds of teachers working in a variety of situations.

Which Category Do You Fit?

- ☐ Small Schools Teacher
 - ☐ inexperienced
 - ☐ experienced, but in other subject areas
 - ☐ experienced in teaching Mathematics 9, but wanting to try a different approach
- ☐ Distance Learning Teacher
 - ☐ travelling to schools within the jurisdiction
 - ☐ using facsimile and teleconferences to teach students within the area
- ☐ Larger Schools Teacher
 - ☐ inexperienced
 - ☐ experienced in teaching Mathematics 9, but wanting to try a different approach



Because these materials have been created by experienced classroom teachers and distance learning specialists, they have many advantages for students and teachers regardless of their situations.

Advantages for Students

- incorporates a strong learner-centred philosophy
- promotes such qualities in the learner as autonomy, independence, and flexibility
- is developed through media which suit the needs and circumstances of the learner
- reflects the experiential background of Alberta students
- opens up opportunities by overcoming barriers that result from geographical location
- promotes individualized learning, allowing learners to work at their own pace

Advantages for Teachers

- allows teachers maximum teaching time and minimizes preparation time
- includes different routes through the materials to suit different learners
- incorporates a wide range of teaching strategies, in particular those using independent and individual learning
- delivers curriculum designed by education specialists that reflects the Alberta Education Program of Studies with an emphasis on Canadian content
- provides learning materials which are upwardly compatible with advanced educational technology

Does it sound like something you could use?

This Learning Facilitator's Manual begins with an overview of the current Alberta Education Program of Studies for Mathematics 9. This summary is included for inexperienced teachers or those teachers who have found themselves teaching Mathematics 9 when their training is in other subject areas. This brief summary is not meant to replace the Alberta Education Program of Studies, but rather to help teachers confirm the highlights of the program.

Other parts of this introduction have also been included to help teachers become familiar with this new learning package and determine how they might want to use it in their classroom.

Beyond the introduction the guide itself contains answers, models, explanations, and other tips generated by the teachers who authored this course.

The module booklets, assignment booklets, and LFMs are the products of experienced classroom teachers and distance learning specialists. It is the hope of these teachers that their experience can be shared with those who want to take advantage of it.



Overview of the Program of Studies

Rational Philosophy

This program was designed and developed using these assumptions:

- It is important to enhance a student's ability to solve problems.
- In learning new concepts students need to progress in stages from concrete, through pictorial, to symbolic.
- Students have varying learning styles and abilities which must be recognized.
- Calculators and computers are tools with which students must become familiar in order to function effectively in this technological age.
- Students need opportunities to practise new skills and maintain previously developed skills.
- The progress students make in meeting mathematics objectives should be evaluated using both formal and informal methods.

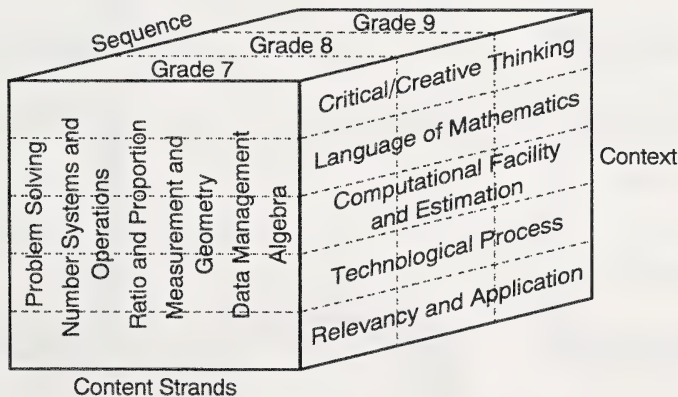
Goals

The goals of this program are to enable students to do the following:

- to grow in their capability to solve problems
- to use mathematics as a tool in the pursuit of personal goals and aspirations
- to develop good self-concepts and positive attitudes towards mathematics and lifelong learning

Program Dimensions

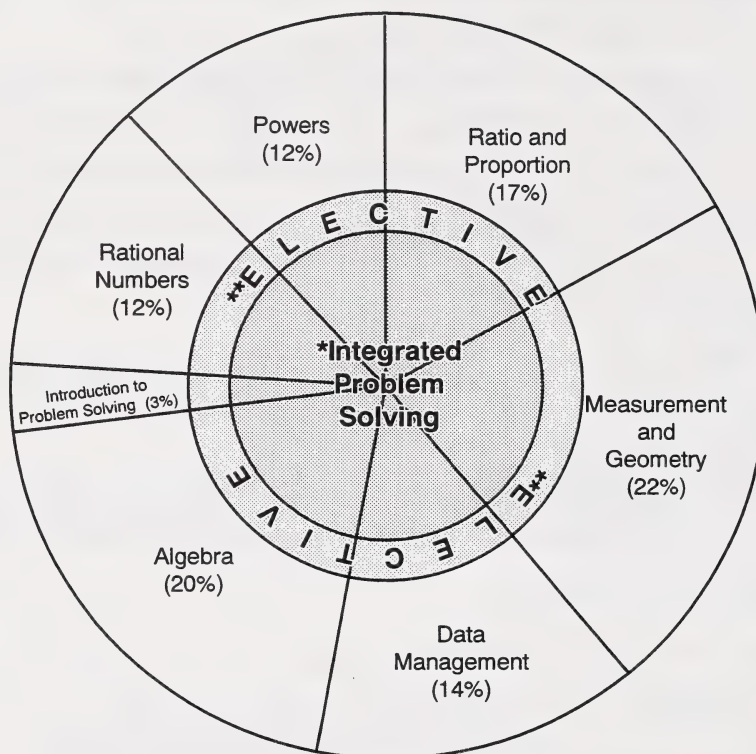
The dimensions of the Junior High Mathematics Program can be summarized by the following diagram:



The Junior High Mathematics Program has two components: the required component and the elective component. Eighty percent of the student's time shall be spent on the basic skills, knowledge, and attitudes outlined above. Twenty percent of the student's time shall be spent on activities meeting individual needs (remedial and/or enrichment). The enrichment shall be horizontal and not vertical; it is not intended to provide acceleration.

Suggested Time Allocations

The minimum requirement for the program is 100 hours. The breakdown of time is shown by the following diagram:

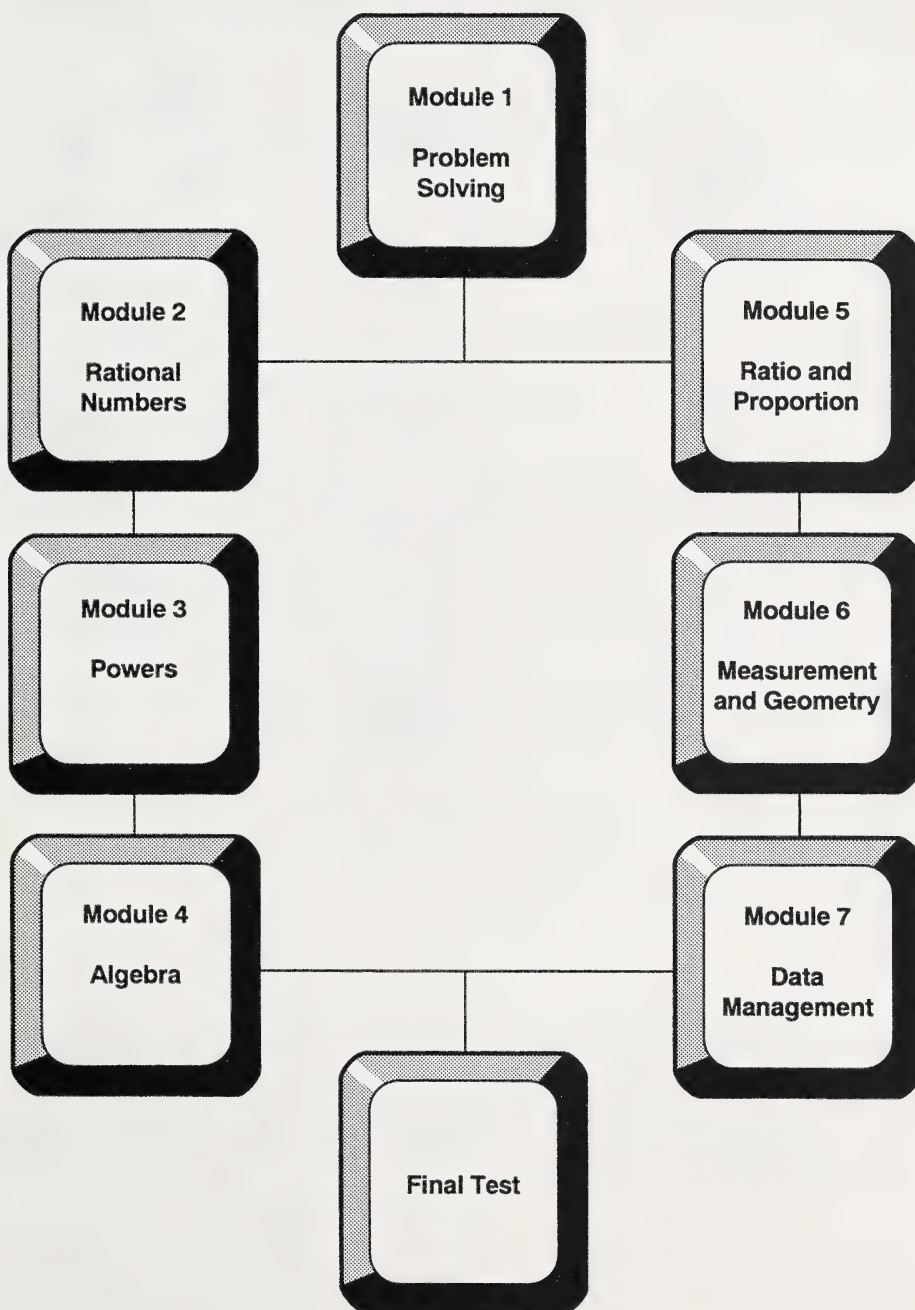


* Integrated Problem Solving – 20% of time

** Integrated Elective (meeting individual needs) – 20% of time

Overview of Mathematics 9

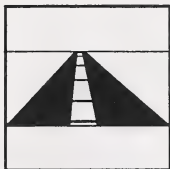
Mathematics 9 has seven modules and a final supervised test.



Symbols and Terms Used in This Course

Symbols

Mathematics 9 has a number of symbols in the margins. These symbols are used throughout the course.



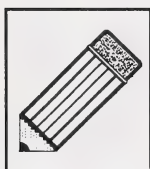
WHAT LIES
AHEAD



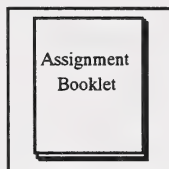
WORKING
TOGETHER



PRACTICE
ACTIVITY



PRINT
ALTERNATIVE



ASSIGNMENT
BOOKLET



COMPUTER
ALTERNATIVE



CALCULATOR

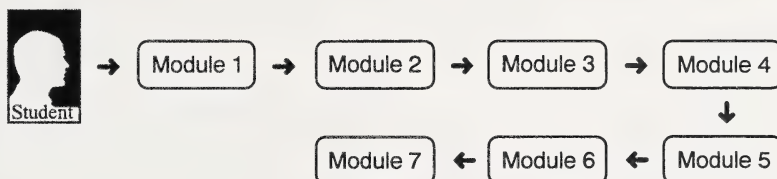
Be sure that you and the student become familiar with these symbols.

Terms

When new terms or concepts are introduced in the course, it is important that the student understands that the terms are new and that they should read carefully to fully understand what the terms mean. The course has been designed so that new terms or concepts are given special attention. They are **defined** and **explained** within the sentence or paragraph. Also check the beginning of the appendix in each module booklet. A special glossary is included. As you supervise, make sure the student knows what the new term and concepts are and what they mean.

Sequencing of Mathematics 9

It is recommended that you start with Module 1 because this module includes basic introductory information. It is further recommended that you complete the modules in order because each module requires skills introduced in previous modules.



These materials are very comprehensive. It is important that you assure the students that the learning package is designed to build on what the student already knows. Students are not expected to do all the activities. Explain that you will help each student decide which activities are appropriate to his or her level of understanding and learning-style preferences.

There are built-in pretests in Modules 2 to 7. These will help you and the student to recognize his or her strengths and weaknesses and to individualize the program. If the student already has a good understanding of the concepts to be taught or reviewed in a section, most of the section can be omitted. (However, the student is encouraged to skim the section and do a few sample questions.) If the student's knowledge of the concept to be taught or reviewed is weak, all the necessary information is provided. (The student is then encouraged to work through the entire section carefully and do most of the questions.)

These materials attempt to make mathematics real and fun. Every attempt has been made to give students a variety of meaningful activities and to give the students a chance to know when they have achieved mastery.

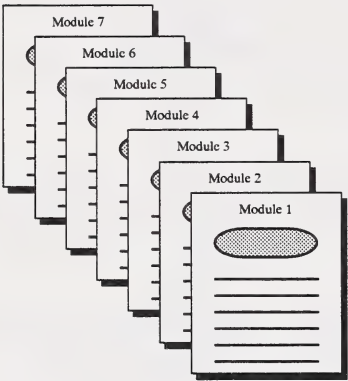
These materials also have integrated videos, computer software, and learning aids to add variety and cater to learning-style preferences. Therefore, this learning package is more than just workbooks; it is a complete program.

Structure of the Learning Package

Basic Design

This new learning package involves many other components in addition to the Learning Facilitator’s Manual.

Modules



The print components involve many booklets called modules. These modules contain guided activities that instruct students in a relevant, realistic setting.

The modules have been specially designed to promote such qualities in the learner as autonomy, independence, and flexibility. Writers have incorporated such teaching strategies as working from the concrete to the abstract, linking the old to the new, getting students actively involved, and using advance, intermediate, and post organizers. Many other techniques enable learners to learn on their own for at least some of the time.

Contents at a Glance
Module Introduction
Section 1 What Lies Ahead Practice Activities etc.
Section 2 What Lies Ahead Practice Activities etc.
Section 3 What Lies Ahead Practice Activities etc.
Section 4 What Lies Ahead Practice Activities etc.
Module Conclusion

The structure of the module booklets follows a systematic design that is illustrated on the left. The modules have many helpful features. Module 1 introduces problem solving, and this skill is emphasized throughout the package. Pretests are included to help you determine the student’s strengths and weaknesses in each strand. Background material is included for students who have not mastered previously developed skills, so they can make a smooth transition. New skills are developed carefully and fully, and there is an abundance of practice. Games, manipulative activities, and math trivia are included to make mathematics fun and interesting.

The module is made up of two or more closely related sections. The sections begin with a list of the skills to be learned and follows with activities that develop skills and knowledge centred around a theme.

The activities may involve print, audio, video, computer, or laser videodisc formats. At times the student and the learning facilitator are allowed to choose the activity that best suits the student’s needs and interests. This flexibility caters to each student’s personal situation.

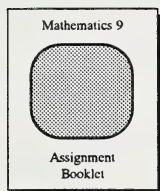
The module conclusion focuses on the skills and strategies that the student has learned.

Note

The modules are very extensive in order to meet the needs of all students.

Students are **not** expected to do all the activities in the module booklets. You will help each student decide which activities are appropriate for his or her level of understanding and learning-style preferences.

Assignment Booklet



Accompanying each module is an assignment booklet. The activities in these booklets can be used for formative and for summative assessments. The students should complete these assignment booklets when they have thoroughly reviewed the module materials. The assignment booklets have been designed for classroom use, for faxing, or for mailing. **If the booklets are not being mailed, you should remove the outside cover.**

Media



COMPUTER DISK



VIDEOCASSETTE



COURSE
AUDIOCASSETTE
(providing general
teacher guidance)

The package also includes references to media. Some types of media such as computer disks and laser videodisks are optional choices for students; however, there are activities that require students to view certain videos. These mandatory videos are listed on the following page. It is important that you acquire these videos as you are planning the course. In addition to the mandatory videos, optional videos have been mentioned at various points in the modules. A list of the optional videos is also included on the following page. More information about the videos can be found within the LFM.


A special audiocassette features a teacher guiding the student through the course. The appearance of the teacher icon reminds students that there is this additional help available. If the students are working individually, you may find this cassette a valuable asset. If you are working in a large group, you may wish to guide the students yourself.

Equipment

Students require a scientific calculator and a geometry set.

Materials, Media, and Equipment

Mandatory Components

Equipment (Hardware)	Media	Materials
<ul style="list-style-type: none"> scientific calculator with  VCR and TV 	<ul style="list-style-type: none"> Mandatory Video List: <ul style="list-style-type: none"> <i>MATH WORKS: Identifying the Problem, SOLVE IT: Reasonableness of Answers, SOLVE IT: Guess-Check-Revise, MATH WORKS: Simplifying the Problem, SOLVE IT: Solving a Simpler Problem, SOLVE IT: Using Logical Reasoning, MATH MOVES: Number Theory, MATH MOVES: Equations – Solving With One Step, MATH MOVES: Equations – Solving With More Than One Step, MATHWAYS: The Percent</i> 	<ul style="list-style-type: none"> LFM for Mathematics 9 one complete set of module booklets (7) and assignment booklets (7) for each student There is a final test.

Videocassettes or laser videodiscs used in the course may be available from the Learning Resources Distributing Centre or ACCESS Network. You may also wish to call your regional library service for more information. Computer programs may be available from your regional media centre, the Learning Resources Distributing Centre, ACCESS Network, or a computer software supplier.

Optional Components

Equipment (Hardware)	Media	Materials
<ul style="list-style-type: none"> audiocassette player prepared audiocassettes (come with learning package) Apple™ or compatible computer VAX™ computer (required to access the Learning Management System) 	<ul style="list-style-type: none"> Optional Video List: <ul style="list-style-type: none"> <i>MATH MOVES: Integers, MATH MOVES: Adding and Subtracting Fractions, MATH MOVES: Multiplying and Dividing Fractions, Geometric Constructions, MATH WORKS: Sampling, SOLVE IT: Sampling</i> computer programs The Learning Management System (LMS) (available from the Alberta Distance Learning Centre) 	

Using This Learning Package in the Classroom

Conventional Classroom

Whether your classroom has desks in rows or tables in small groups, you may be most comfortable with a learning system that you can use with all your students in a paced style. In other words, you may want a package that will suit all of your students, so they can move through the materials as one group or several small groups. Because these materials contain different routes or pathways within each module, they can address various learning styles and preferences. The materials also include many choices within the activities to cater to different thinking levels and ability levels. Because of their versatility and flexibility, these materials can easily suit a conventional classroom.

Open-Learning Classroom

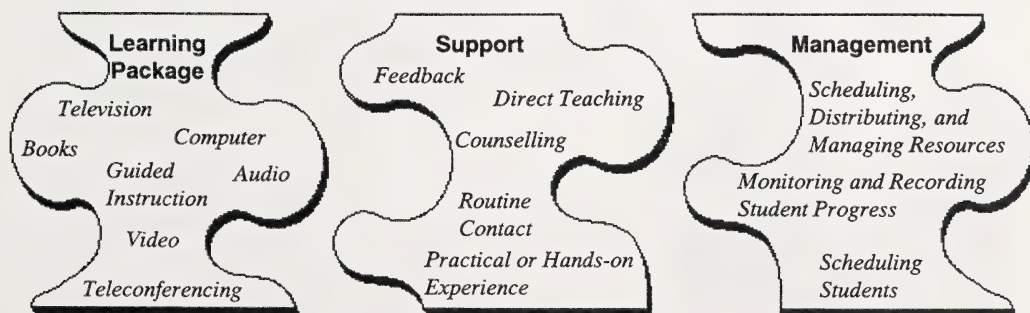
Open learning is the concept of opening up opportunities by overcoming barriers of time, pace, and place by giving the learners a package specially designed to enable them to learn on their own for at least some of the time.

Such a concept is not new. Many teachers can recite attempts to establish an individualized learning system as they recognized the importance of trying to personalize courseware to meet each individual student's needs. But these efforts often failed due to lack of time and lack of quality materials that conformed to Alberta specifications.

Due to advanced educational technology and improved Alberta-specific learning packages, a student-centred approach is now possible. Improved technology now allows us to provide support to learners individually, regardless of their pace or location. A teacher cannot be in twenty-eight places at one time offering guidance. However, media and a well-designed learning package can satisfy individual needs. Technology can also help provide an effective management system needed to track the students as they progress independently through the materials.

The key to a successful open-learning system depends on three vital elements: a learning package specially designed to enable students to learn effectively on their own for at least some of the time; various kinds of learner support; and a management system and style that ensures that the open-learning system runs smoothly.

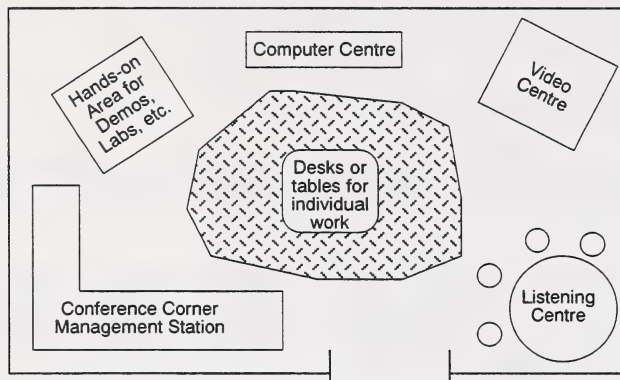
The Key to a Successful Open-Learning System



Learning Package

The specially designed learning package needed for a successful open-learning system has been developed for you. The objectives teach current Alberta specifications using strategies designed for individualized instruction. As the learning facilitator, you need to be sure to have all the components in the learning package available to students as needed.

If adequate numbers of media are available to satisfy the demand, a centre can be established for specific media.



You may not have the luxury to have enough hardware to set up a permanent video or computer centre in your classroom. In that case, students should be encouraged to plan ahead. Perhaps every three to five days they should preview their materials and project when they would need a certain piece of media. This would allow you to group students, if necessary, or reserve media as required.

Support

Support is definitely a key element for successful learning, and when you're planning an individualized, non-paced program, you need to carefully plan when and how support will be given.

The materials contain a form of consistent support by providing immediate feedback for activities included in the module booklet. Solutions, models, explanations, and guides are included in the appendix of every module booklet. These are included so students can receive immediate feedback to clarify and reinforce their basic understanding before they move on to higher levels of thinking. The answers, explanations, and examples for each of the module assignments and pretests are included in this LFM.

As the learning facilitator, you may be needed to offer more personal guidance to those students having difficulty, or you may need to reinforce the need for students to do these activities carefully before attempting the assignments in the assignment booklet.

The activities include choices and pathways. If a student is having difficulty, you may need to encourage that student to work on all the choices rather than one. This would provide additional instruction and practice in a variety of ways.

Another form of support is routine contact with each individual. This might be achieved with a biweekly conference scheduled by you, or as students reach a certain point (e.g., after each section is completed), they may be directed to come to the conference area.

Special counselling may be needed to help students through difficult stages. Praise and encouragement are important motivators, particularly for those students who are not used to working independently.

Direct teaching may be needed and scheduled at certain points in the program. This might involve small groups or a large group. It might be used to take advantage of something timely (e.g., election, eclipse, etc.), something prescheduled like the demonstration of a process, or something involving students in a hands-on, practical experience.

Support at a distance might include tutoring by phone, teleconferencing, faxing, or planned visits. These contacts are the lifeline between learners and distance education teachers, so a warm dialogue is essential.

Management

Good management of an open-learning system is essential to the success of the program. The following areas need action to ensure that the system runs smoothly:

- **Scheduling, Distributing, and Managing Resources** – As discussed earlier, this may require a need for centres or a system for students to project and reserve the necessary resources.
- **Scheduling Students** – Students and teachers should work together to establish goals, course completion timelines, and daily timelines. Although students may push to continue for long periods of time (e.g., all morning), teachers should discourage this. Concentration, retention, and motivation are improved by taking scheduled breaks.
- **Monitoring Student Progress** – You will need to record when modules are completed by each student. Your data might also include the projected date of completion if you are using a student contract approach.



Sample of a Student Progress Chart

Mathematics 9		Module 1	Module 2	Module 3	Module 4	Module 5	Module 6	Module 7	Final Test
<i>Billy Adams</i>	P								
	A								
<i>Louise Despina</i>	P								
	A								
<i>Violet Klaissian</i>	P								
	A								
P = Projected Completion Date A = Actual Completion Date									

The student could keep a personal log as well. Such tracking of data could be stored easily on a computer.

- Recording Student Assessments – You will need to record the marks awarded to each student for work completed in each module assignment booklet. The marks from these assignment booklets will contribute to a portion of the student's final mark. Other criteria may also be added (a special project, effort, attitude, etc.). Whatever the criteria, they should be made clear to all students at the beginning.

Sample of a Student Assessment Chart

Mathematics 9	Module 1	Module 2	Module 3	Module 4	Module 5	Module 6	Module 7	Year's Average	Final Test	Final Mark
<i>Billy Adams</i>	67	65	54	47	78	67	65	63		
<i>Louise Despina</i>	43	50	54	55	48	42	48	49		
<i>Violet Klaissian</i>	65	65	66	68	67	70	68	67		

Letter grading could easily be substituted.

- Recording Effectiveness of System – Keep ongoing records of how the system is working. This will help you in future planning.

Sample of a System Assessment Chart

Module 1			
Date	Module Booklet	Assignment Booklet	Resources/Media

The Role of the Teacher in an Open-Learning Classroom

The teachers in a conventional classroom spend a lot of time talking to large groups of learners. The situation in open learning requires a different emphasis. Teachers will probably meet learners individually or in very small groups.

With this approach it is necessary to move beyond the idea of a passive learner depending largely on a continually supportive teacher. The teacher must aim to build the student's confidence, to stimulate the learner into self-reliance, and to guide the learner to take advantage of routes that are most meaningful and applicable to the learner.

These materials are student-centred, not teacher-centred. The teacher needs to facilitate learning by providing general support to the learner.

Evaluation

Evaluation is important to the development of every learner. Data gathering and processing, and decision making, at the student and teacher level, serve as means of identifying strengths and weaknesses.

These specially designed learning packages contain many kinds of informal and formal evaluation.

Observation

In the classroom the teacher has the opportunity to see each student perform every day and to become aware of the level and nature of each student's performance.

Observations are more useful if they are recorded in an organized system. The following list of questions is a sample of types of observations and how they can be collected.

Observation Checklist

1. Does the student approach the work in a positive manner?
2. Is the student struggling with the reading level?
3. Does the student make good use of time?
4. Does the student apply an appropriate study method?
5. Can the student use references effectively, etc.?

B. Adams	L. Despina	V. Klaisian	H. Smith	K. Dalley

Observation may suggest a need for an individual interview with a student.

Individual Conferences

Individual conferences may be paced (scheduled) by the calendar, at certain points in the module, or they may be set up only as needed or requested.

During these conferences teachers can determine the student's progress and can assess the student's attitudes toward the subject, the program, school, and self, as well as the student's relationship with other students. With guided questions the teacher can encourage oral self-assessment; the student can discuss personal strengths or weaknesses in regard to the particular section, module, or subject area.

Self-Appraisal

Self-appraisal helps students recognize their own strengths and weaknesses. Through activities that require self-assessment, students also gain immediate feedback and clarification at early stages in the learning process. Teachers need to promote a responsible attitude toward these self-assessment activities. Becoming effective self-assessors is a crucial part of becoming autonomous learners. By instructing, motivating, providing positive reinforcement, and systematically supervising, the learning facilitator will help students develop a positive attitude toward their own progress.

For variation, students may be paired and peer-assessing may become part of the system. The teacher may decide to have the student self-assess some of the activities, have a peer assess other activities, and become directly involved in assessing the remainder of the activities.

When the activities have been assessed, the student should be directed to make corrections. This should be made clear to students right from the start. It is important to note the correct association between the question and the response to clarify understanding, aid retention, and be of use for study purposes.

Many of the activities include choices for the student. If the student is having difficulty, more practice may be warranted, and the student may need to be encouraged to do more of the choices.

Self-appraisal techniques can also be introduced at the individual conferences. Such questions as the following might be included:

- What steps are you taking to improve your understanding of this topic?
- What method of study do you use most?
- How do you organize your material to remember it?
- What steps do you follow when doing an assignment?
- What could you do to become an even better reader?
- Do you have trouble following directions?
- Did you enjoy this module?

A chart or checklist could be used for recording responses.

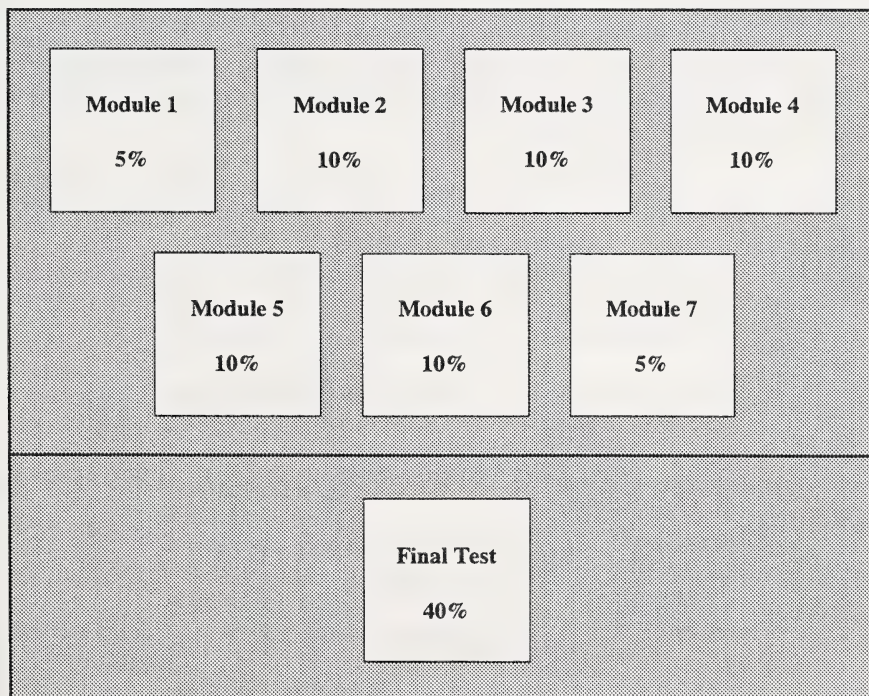
Informal Evaluation: Assignments

Informal evaluation, such as the assignments included in each module, are an invaluable aid to the teacher. They offer ongoing assessment information about the student's achievement and the behaviour and attitudes that affect that achievement.

Each module contains a separate booklet called the Assignment Booklet. This booklet assesses the knowledge or skills that the student has gained from the module. **The student's mark for the module may be based solely on the outcome of learning evident in the assignment booklet; however, you may decide to establish a value for other variables such as attitude or effort.** It is important that you establish at the beginning which outcomes will be evaluated, and that all students clearly understand what is expected.

Final Test

All LFM's include a formal final test which can be photocopied for each member of the class. The test, closely linked to the learning outcomes stated in the module booklets, gives the teacher precise information concerning what each student can or cannot do. Answers, explanations, and marking guides are also included. The value of the final test and each module is the decision of the classroom teacher. Following is a suggestion only.



Introducing Students to the System

Your initiation to these learning materials began with a basic survey of what was included and how the components varied. This same process should be used with the class. After the materials have been explored, a discussion might include the advantages and the disadvantages of learning independently or in small groups. The roles of the students and teacher should be analysed. The necessary progress checks and rules need to be addressed. Your introduction should motivate students and build a responsible attitude toward learning autonomously.

Skill Level

It is important for students to understand that there are certain skills that they will need in order to deal successfully with the course materials. They are listed below:

- understanding and using instructional materials (table of contents, appendix, and glossary)
- interpreting maps, graphs, and charts
- using reference materials
- recognizing special symbols
- using a scientific calculator

Other general skills are using reliable study methods, outlining, and learning to read at a flexible rate.

To decide the level and amount of instruction needed to accommodate the varied levels among students, you may wish to prepare and administer skill inventories or pretests. If most students need help with a particular skill, you may want to plan a total class instructional session. If only certain students lack a skill, you may want to set up a temporary skill group to help students who need it, or you may want to develop a skills file for this purpose.

Reading Level

These course materials are largely print based, but poorer readers need not be discouraged. It is important that you assure the students that these materials have been designed for easy reading. The authors have employed special strategies that lower and control the reading level. Some of them are

- the conscious selection of vocabulary and careful structuring of sentences to keep the materials at an independent reading level
- the integration of activities, examples, and illustrations to break text into appropriate-sized chunks
- the inclusion of many kinds of organizers (advance, graphic, intermediate, concept mapping, post organizers) to help give students a structure for incorporating new concepts
- the recognition that vocabulary and concepts are basic to understanding content materials and, thus, must be handled systematically (defined in context, marginal notes, footnotes, and often in a specialized glossary)

- the acknowledgement that background knowledge and experience play a vital role in comprehension
- the systematic inclusion of illustrations and videos to help poorer readers and visual learners, and audiocassettes and software as an alternative to print-based learning
- a variety of formats (paragraphs, lists, charts, etc.) to help poorer readers who do not absorb or retain main ideas easily in paragraph format
- the inclusion of media and activity choices to encourage an active rather than passive approach
- instruction in a meaningful setting rather than in a contrived, workbook style
- using purposeful reading, viewing, and doing to produce better interpretation of the course materials
- the recognition that students need structured experiences when reading, viewing, or listening to instructional materials: developing pupil readiness, determining the purpose, providing guided instruction and feedback, rereading if necessary, and extending (This structure closely resembles the reading process.)

To help make the learning package more readable, you can begin your module preparation by reading (viewing, listening to) all the related materials that are going to be used. You need a solid background in order to assess and develop a background knowledge for students. The students' experiential bases may be assessed through brainstorming sessions concerning the topic, or by using visuals and guided questions to predict what the topic might be about.

Suggested Answers

Module 1: Problem Solving

Gathering Materials

For this module the student will need the following items available from the Learning Resources Distributing Centre.

- Student Module 1: Problem Solving
- Assignment Booklet for Module 1
- optional course audiocassette for Module 1
- required videocassettes for Module 1 (*MATH WORKS: Identifying the Problem*, *SOLVE IT: Reasonableness of Answers*, *MATH WORKS: Simplifying the Problem*, *SOLVE IT: Solving a Simpler Problem*, *SOLVE IT: Using Logical Reasoning*)

Students may use the following computer programs if they have access to them and an Apple™ or compatible computer.

- *Problem Solving Strategies* (MECC)
- *Math Strategies: Solving Problems* (SRA)
- *Mathematics Activity Courseware 7*, Disk A (Houghton-Mifflin.)

In addition, the students will need a binder and loose-leaf paper for their responses, a scientific calculator with a $\frac{b}{a}$ key, and a geometry set.

Hold the Assignment Booklet for safekeeping until the student is ready for it.

Guiding the Student

Each section of Module 1 deals with a different aspect of problem solving. Students will learn about each aspect by viewing a video program and/or reading notes in the student module and practising problems. An optional course audiocassette will help you guide the student through the module.

Ensure that the student reads the welcome pages and the module introduction. Help the student plan a schedule. An average student spends about three weeks of a 40-week school year on Module 1.

Use the guidelines in the beginning of this guide to monitor the student's progress and provide encouragement.

Refer to the suggested answers in the Appendix of Student Module 1: Problem Solving when guiding the student through the Practice Activities in Module 1.

Assignment Answer Key

Part 1: Short-Answer Questions

1. a. A problem is a task for which the method for finding the answer is not immediately known.
- b. The four stages to solve any problem are
 - understanding the problem
 - developing a plan
 - trying the plan
 - looking back

2. ~~Canada's thirteenth prime minister was John Diefenbaker. Diefenbaker's path to the prime minister's office was long. He ran federally for Prince Albert in 1925 and 1926, provincially in 1929 and 1938, and for mayor of Prince Albert in 1933. He lost each time. In 1940 he was elected a member of Parliament for the first time. In 1956 he became leader of the Progressive Conservative party, and in 1957 he became prime minister. In the 1963 election, the Liberals returned to power. Diefenbaker, however, remained in politics until his death in 1963. How long was Diefenbaker prime minister?~~

John Diefenbaker became prime minister in 1957. In the 1963 election, the Liberals returned to power. How long was Diefenbaker prime minister?

3. Answers will vary. Here is an example.

A shopkeeper bought an antique for \$7250 then sold it for \$10 000. How much profit did the shopkeeper make on the antique?

4. a. B

b. $4 \times 589.8 = 2359.2$

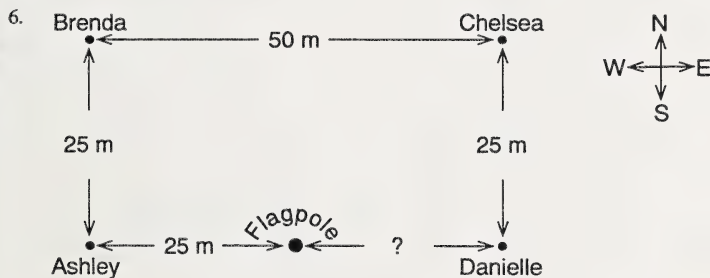
$90 \times 24.7 = 2223$

5. $96 - 12 = 84$

Each of the 84 wagons has to carry one more person.

$84 \div 12 = 7$

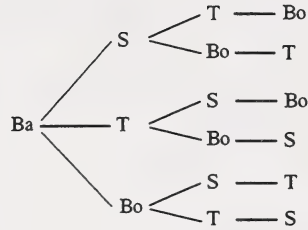
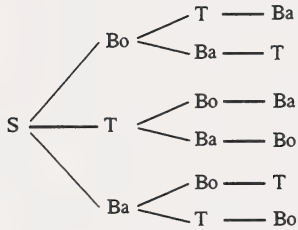
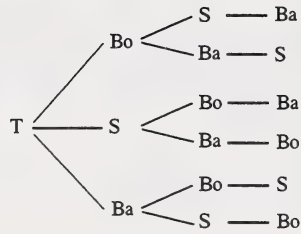
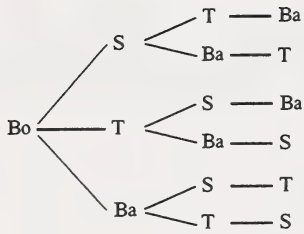
So, each wagon had 7 people originally.



$50 \text{ m} - 25 \text{ m} = 25 \text{ m}$

Danielle was 25 m due east of the flagpole.

7.



8.

	Wins	Losses	Test
Guess 1	50	20	$50 + 20 \neq 150$
Guess 2	55	25	$55 + 25 \neq 150$
Guess 3	60	30	$60 + 30 \neq 150$

9. a.

Stops	Passengers Getting Off the Subway Train
1	1
2	2
3	4
4	8
5	16

Pattern

$\left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \times 2$
 $\left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \times 2$
 $\left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \times 2$
 $\left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \times 2$

At the fifth stop, 16 passengers will get off.

b.

Stops	Passengers Getting Off	Passengers Left
1	1	126
2	2	124
3	4	120
4	8	112
5	16	96
6	32	64
7	64	0

Seven stops are required before all the passengers are off.

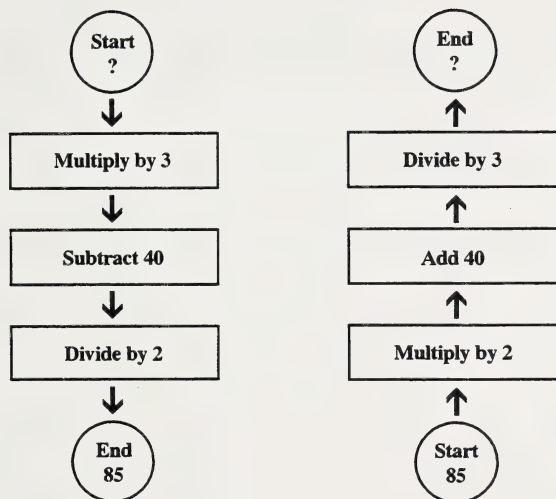
10. a. $100 - 20 = 80$
 $80 \div 4 = 20$

Each person received \$20.

b. $378\,273 - 13\,598 = 364\,675$
 $364\,675 \div 29 = 12\,575$

Each grandchild received \$12 575.

11. Flow Chart



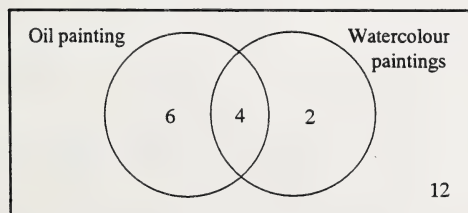
The original number is 70.

12.

	Art	Bill	Clive
Red	x	✓	x
Blue	x	x	✓
Green	✓	x	x

Art chose green, Bill chose red, and Clive chose blue.

13.



Twelve students do not like to do either oil or watercolour paintings.

Part 2: Problems

1.

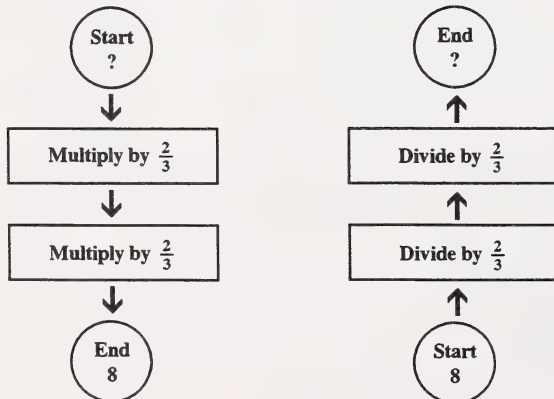
People	Handshakes	Pattern
1	0	
2	1	} +1
3	3	} +2
4	6	} +3
5	10	} +4
6	15	} +5
7	21	} +6
8	28	} +7
9	36	} +8

If 36 handshakes are exchanged, there are 9 people at the party.

2. Answer will vary. Here is one possibility.

X	X	X	X	X	X
X	X	X		X	
	X	X		X	X
	X	X	X	X	

3. Methods will vary. Here is one method.



There were 18 prunes in the bowl originally.

4. Methods will vary. Here is one method.

Write the two-digit numbers in which the first digit is greater than 6 and the second digit is odd. Then circle the ones divisible by 3 that do not have two digits that are the same.

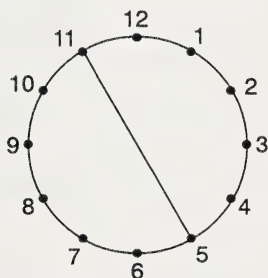
71, 73, (75), 77, 79, (81), 83, 85, (87), 89, 91, (93), 95, 97, 99

Rapheal is one of these ages: 75, 81, 87, or 93.

5. Methods will vary. Students can use objects or diagrams and logical thinking.



6. Methods will vary. Students can use objects or diagrams and logical thinking.



There are 12 children in the circle.

7. Methods will vary. Students can use the guess-check-revise method and logical reasoning.

- The number must be less than 72.
- The number must be odd.
- The number cannot be 5, 15, 25, 35, 45, 55, or 65.

	Number	Test
Guess 1	71	$71 \div 5 = 14 \text{ R}1$
Guess 2	67	$67 \div 5 = 13 \text{ R}2$
Guess 3	63	$63 \div 4 = 15 \text{ R}3$, $63 \div 5 = 12 \text{ R}3$, $63 \div 6 = 10 \text{ R}3$

The number of eggs is 63.

8. Students can act out the problem, use objects, or make a diagram.

	Locker Number																								
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Students	1	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O
	2		C		C		C		C		C		C		C		C		C		C		C		C
	3			C			O			C			O			C			O			C			O
	4				O			O				C				O				O					C
	5					C				O					O					C					C
	6						C					O						C						O	
	7							C						O							O				
	8								C							C								C	
	9									O								O							
	10										C									O					
	11											C										O			
	12												C											O	
	13													C											
	14														C										
	15															C									
	16																O								
	17																	C							
	18																		C						
	19																			C					
	20																				C				
	21																					C			
	22																						C		
	23																							C	
	24																								C
	25																								O

Twenty doors are closed. All but the first, fourth, ninth, sixteenth, and twenty-fifth are closed.

Module 2: Rational Numbers

Gathering Materials

For this module the student will need the following items available from the Learning Resources Distributing Centre.

- Student Module 2: Rational Numbers
- Assignment Booklet for Module 2
- optional course audiocassette for Module 2

Students may view the following video programs if they have access to them.

- *MATH MOVES: Integers* (ACCESS Network)
- *MATH MOVES: Adding and Subtracting Fractions* (ACCESS Network)
- *MATH MOVES: Multiplying and Dividing Fractions* (ACCESS Network)

Students may use the following computer programs if they have access to them and an Apple™ or compatible computer.

- *Integers Fast Facts* (Edusoft)
- *Conquering Fractions* (MECC)
- *Computer Drill and Instruction: Mathematics, Level D* (SRA)

In addition, the students will need a binder and loose-leaf paper for their responses, a scientific calculator with a $\boxed{\frac{a}{b}}{\frac{c}{d}}$ key, and a geometry set.

Hold the Assignment Booklet for safekeeping until the student is ready for it.

Guiding the Student

Each section of Module 2 deals with a different aspect of operations with rational numbers. Sections 2 to 12 review previously developed skills. Sections 13 to 15 develop new skills. The Pretest in Section 1 will help you discover the student's strengths and weaknesses and make a personal study plan.

Ensure that the student reads the welcome pages and the module introduction, and does Section 1. Then check the answers to the Pretest (suggested answers follow). Help the student develop a personal study plan and schedule. An average student spends about six weeks on Module 2.

Use the guidelines provided at the beginning of this guide to monitor the student's progress and provide encouragement.

Only the suggested answers for the Pretest for Module 2 are provided in this guide. Refer to the suggested answers in the Appendix of Student Module 2: Rational Numbers when guiding the student through the Practice Activities in Module 2.

Section 1: Pretest

- +300
 - 21
 - 6700
 - +6
 - 14
- +46
 - 198
 - +230
 - 99
- $49 > 41$
 - $-49 < -41$
 - $7 > -7$
 - $-2 < 1$
 - $-31 < -30$
- 20, -14, -2, 0, +7, +14
 - 1000, -36, -20, -1, +8, +9, +100

5. a. +9 b. -12 c. +5 d. -4
6. a. +2 b. +64 c. -45 d. +18
7. a. +20 b. -21 c. -32 d. +60
8. a. +23 b. -2 c. -8 d. +5
9. a. $(-5) - (-12) = (-5) + (+12)$
 $= +7$
 The temperature has risen 7°C .
- b. $6194 - (-86) = 6194 + 86$
 $= 6280$
 The difference in the heights is 6280 m.
- c. $(-13) + (-10) + (-6) = -29$
 The coldest temperature recorded overnight was -29°C .
- d. $2 \times (-7) = -14$
 Kim had 14 penalty points, or -14 .
- e. $(-1800) \div (-30) = 60$
 It will take 60 min to descend to -1800 m at this rate.
10. $\frac{5}{12}$ of the carton of eggs is full.
11. a. $\frac{7}{9} = \frac{14}{18} = \frac{21}{27} = \frac{28}{36} = \frac{35}{45} = \frac{42}{54} \dots$ b. $\frac{14}{21} = \frac{2}{3}$ c. $\frac{75}{13} = 5\frac{10}{13}$ d. $2\frac{1}{8} = \frac{17}{8}$
- e. $\frac{4}{125} = \frac{32}{1000}$ or $\frac{4}{125} = 4 \div 125$ f. $\frac{5}{9} = 5 \div 9$
 $= 0.032$ $= 0.032$ $= 0.555\dots$
 $= 0.5$ or $0.\overline{5}$
- g. $0.135 = \frac{135}{1000}$
 $= \frac{27}{200}$
- h. $\frac{9}{14}$ will produce a repeating decimal.
12. $\frac{5}{8} = 0.625$ $\frac{5}{8} = \frac{25}{40}$
 $\frac{7}{10} = 0.7$ $\frac{7}{10} = \frac{28}{40}$
 or
 $\frac{3}{4} = 0.75$ $\frac{3}{4} = \frac{30}{40}$
 $\frac{11}{20} = 0.55$ $\frac{11}{20} = \frac{22}{40}$
- So, the descending order is $\frac{3}{4}$, $\frac{7}{10}$, $\frac{5}{8}$, and $\frac{11}{20}$.
13. Estimates will vary. Exact answers are given.
- a. $\frac{5}{8} + \frac{2}{8} = \frac{7}{8}$ b. $\frac{3}{6} - \frac{1}{6} = \frac{2}{6}$
 $= \frac{1}{3}$

$$\begin{aligned} \text{c. } 1\frac{1}{10} + 2\frac{3}{10} &= 3\frac{4}{10} \text{ or } 1\frac{1}{10} + 2\frac{3}{10} = \frac{11}{10} + \frac{23}{10} \\ &= 3\frac{2}{5} &= \frac{34}{10} \\ &= 3\frac{4}{10} \\ &= 3\frac{2}{5} \end{aligned}$$

$$\begin{aligned} \text{d. } 2\frac{3}{5} - 1\frac{4}{5} &= 1\frac{8}{5} - 1\frac{4}{5} \text{ or } 2\frac{3}{5} - 1\frac{4}{5} = \frac{13}{5} - \frac{9}{5} \\ &= \frac{4}{5} &= \frac{4}{5} \end{aligned}$$

$$\begin{aligned} 14. \text{ a. } \frac{3}{5} + \frac{1}{4} &= \frac{12}{20} + \frac{5}{20} \\ &= \frac{17}{20} \end{aligned}$$

$$\begin{aligned} \text{b. } \frac{7}{8} - \frac{3}{4} &= \frac{7}{8} - \frac{6}{8} \\ &= \frac{1}{8} \end{aligned}$$

$$\begin{aligned} \text{c. } 1\frac{1}{6} + 2\frac{4}{9} &= 1\frac{3}{18} + 2\frac{8}{18} \text{ or } 1\frac{1}{6} + 2\frac{4}{9} = \frac{7}{6} + \frac{22}{9} \\ &= 3\frac{11}{18} &= \frac{21}{18} + \frac{44}{18} \\ &= \frac{65}{18} \\ &= 3\frac{11}{18} \end{aligned}$$

$$\begin{aligned} \text{d. } 3\frac{3}{10} - 1\frac{2}{5} &= 3\frac{3}{10} - 1\frac{4}{10} \text{ or } 3\frac{3}{10} - 1\frac{2}{5} = \frac{33}{10} - \frac{7}{5} \\ &= 2\frac{13}{10} - 1\frac{4}{10} &= \frac{33}{10} - \frac{14}{10} \\ &= 1\frac{9}{10} &= \frac{19}{10} \\ &= 1\frac{9}{10} \end{aligned}$$

15. The steps in computing mentally will not necessarily be the same as when computing with paper and pencil. Methods for arriving at the answers will vary.

$$\text{a. } 7\frac{1}{4} \quad \text{b. } \frac{1}{2} \quad \text{c. } 7\frac{1}{9} \quad \text{d. } 1\frac{3}{10}$$

16. Estimates will vary. Exact answers are given.

$$\text{a. } \frac{1}{5} \times \frac{1}{6} = \frac{1}{30}$$

$$\text{b. } \frac{3}{7} \times \frac{5}{8} = \frac{15}{56}$$

$$\begin{aligned} \text{c. } 1\frac{3}{4} \times 2\frac{1}{3} &= \frac{7}{4} \times \frac{7}{3} \\ &= \frac{49}{12} \\ &= 4\frac{1}{12} \end{aligned}$$

$$\begin{aligned} \text{d. } 1\frac{2}{3} \times 1\frac{3}{4} &= \frac{5}{3} \times \frac{7}{4} \\ &= \frac{35}{12} \\ &= 2\frac{11}{12} \end{aligned}$$

$$17. \text{ a. } \frac{7}{8} \times \frac{2}{5} = \frac{7}{20}$$

$$\text{b. } \frac{1}{4} \times \frac{2}{3} = \frac{2}{12} = \frac{1}{6}$$

$$\begin{aligned} \text{c. } 1\frac{1}{2} \times 1\frac{5}{6} &= \frac{3}{2} \times \frac{11}{6} \\ &= \frac{33}{12} \\ &= 2\frac{3}{4} \end{aligned}$$

$$\begin{aligned} \text{d. } 2\frac{7}{9} \times 3\frac{3}{5} &= \frac{25}{9} \times \frac{18}{5} \\ &= 10 \end{aligned}$$

18. The steps in computing mentally will not necessarily be the same as when computing with paper and pencil. Methods for arriving at the answers will vary.

a. 14 b. $1\frac{2}{5}$ c. $9\frac{1}{3}$

19. Estimates will vary. Exact answers are given.

a. $\frac{3}{5} \div \frac{1}{5} = 3$ or $\frac{3}{5} \div \frac{1}{5} = \frac{3}{\cancel{5}} \times \frac{\cancel{5}}{1}$
 $= 3$

3 fifths \div 1 fifth = 3

b. $\frac{2}{3} \div 2 = \frac{1}{3}$ or $\frac{2}{3} \div 2 = \frac{\cancel{2}}{3} \times \frac{1}{\cancel{2}}$
 $= \frac{1}{3}$

2 thirds \div 2 = 1 third

c. $\frac{3}{4} \div \frac{3}{8} = \frac{6}{8} \div \frac{3}{8}$ or $\frac{3}{4} \div \frac{3}{8} = \frac{\cancel{3}}{4} \times \frac{\cancel{8}}{\cancel{3}}$
 $= 2$ $= 2$

6 eighths \div 3 eighths = 2

d. $\frac{4}{5} \div \frac{1}{2} = \frac{8}{10} \div \frac{5}{10}$ or $\frac{4}{5} \div \frac{1}{2} = \frac{4}{5} \times \frac{2}{1}$
 $= \frac{8}{5}$ $= \frac{8}{5}$
 $= 1\frac{3}{5}$ $= 1\frac{3}{5}$

e. $1\frac{3}{4} \div \frac{1}{4} = \frac{7}{4} \div \frac{1}{4}$ or $1\frac{3}{4} \div \frac{1}{4} = \frac{7}{4} \div \frac{1}{4}$
 $= 7$ $= \frac{7}{\cancel{4}} \times \frac{\cancel{4}}{1}$
 $= 7$

f. $2\frac{1}{3} \div 1\frac{1}{2} = \frac{7}{3} \div \frac{3}{2}$ or $2\frac{1}{3} \div 1\frac{1}{2} = \frac{7}{3} \div \frac{3}{2}$
 $= \frac{14}{6} \div \frac{9}{6}$ $= \frac{7}{3} \times \frac{2}{3}$
 $= \frac{14}{9}$ $= \frac{14}{9}$
 $= 1\frac{5}{9}$ $= 1\frac{5}{9}$

20. The steps in computing mentally will not necessarily be the same as when computing with paper and pencil. Methods for arriving at the answers will vary.

a. $2\frac{1}{8}$ b. $2\frac{1}{5}$ c. $1\frac{1}{15}$

21. a. $\frac{5}{2}$ b. $\frac{1}{3}$ c. $\frac{3}{4}$

22. Methods for solving this problem will vary.

The numbers are 4 and $\frac{1}{4}$.

$$\begin{aligned}
 23. \quad a. \quad \frac{3}{4} + \frac{1}{3} &= \frac{9}{12} + \frac{4}{12} \\
 &= \frac{13}{12} \\
 &= 1\frac{1}{12}
 \end{aligned}$$

Shelley did the two activities for $1\frac{1}{12}$ h.

$$\begin{aligned}
 b. \quad 14 + 1\frac{3}{4} &= \frac{56}{4} + \frac{7}{4} \quad \text{or} \quad 14 + 1\frac{3}{4} = 14 \div \frac{7}{4} \\
 &= 8 \\
 &= \overset{2}{\cancel{14}} \times \frac{4}{\cancel{7}} \\
 &= 8
 \end{aligned}$$

John can swim 8 lengths in this time at this rate.

$$\begin{aligned}
 c. \quad \frac{1}{8} \times 200 &= \frac{200}{8} \\
 &= 25
 \end{aligned}$$

25 m³ are above the surface of the water.

$$\begin{aligned}
 d. \quad 8\frac{3}{5} - 4\frac{1}{3} &= 8\frac{9}{15} - 4\frac{5}{15} \\
 &= 4\frac{4}{15}
 \end{aligned}$$

Sirius is $4\frac{4}{15}$ light years further from Earth than Alpha Centauri.

24. A rational number is any number that can be expressed as a quotient of two integers.

$$25. \quad a. \quad \frac{7}{8} \qquad b. \quad -0.3 \qquad c. \quad \frac{3}{5}$$

$$\begin{aligned}
 26. \quad a. \quad \frac{1}{4} + \left(-\frac{7}{12}\right) &= \frac{3}{12} + \left(-\frac{7}{12}\right) \\
 &= -\frac{4}{12} \\
 &= -\frac{1}{3} \\
 b. \quad \left(-\frac{1}{3}\right) + \left(-\frac{3}{7}\right) &= \left(-\frac{7}{21}\right) + \left(-\frac{9}{21}\right) \\
 &= -\frac{16}{21} \\
 c. \quad 0.8 + (-0.25) &= 0.55
 \end{aligned}$$

$$\begin{aligned}
 27. \quad a. \quad \frac{3}{10} - \left(-\frac{1}{5}\right) &= \frac{3}{10} + \frac{1}{5} \\
 &= \frac{3}{10} + \frac{2}{10} \\
 &= \frac{5}{10} \\
 &= \frac{1}{2} \\
 b. \quad -5\frac{8}{9} - 1\frac{1}{3} &= -5\frac{8}{9} - 1\frac{2}{9} \\
 &= -6\frac{11}{9} \\
 &= -7\frac{2}{9} \\
 c. \quad 3.5 - 8.2 &= -4.7
 \end{aligned}$$

$$\begin{aligned}
 28. \quad a. \quad -\frac{1}{2} \times \frac{3}{5} &= -\frac{3}{10} \\
 b. \quad \left(-1\frac{3}{4}\right) \times \left(-1\frac{2}{3}\right) &= \left(-\frac{7}{4}\right) \times \left(-\frac{5}{3}\right) \\
 &= \frac{35}{12} \\
 &= 2\frac{11}{12} \\
 c. \quad -0.3 \times 2.4 &= -0.72
 \end{aligned}$$

$$29. \quad a. \quad -\frac{3}{5} \div \frac{1}{5} = -3 \quad \text{or} \quad -\frac{3}{5} \div \frac{1}{5} = -\frac{3}{\cancel{5}^1} \times \frac{\cancel{5}^1}{1} = -3$$

$$b. \quad -1\frac{3}{4} \div \left(-\frac{4}{5}\right) = -\frac{7}{4} \div \left(-\frac{4}{5}\right) \\ = -\frac{7}{4} \times \left(-\frac{5}{4}\right) \\ = \frac{35}{16} \\ = 2\frac{3}{16}$$

$$c. \quad 3.4 \div (-2) = -1.7$$

$$30. \quad a. \quad \frac{3}{7} + \frac{1}{4} + \frac{2}{7} + \frac{3}{4} = \frac{1}{4} + \frac{3}{4} + \frac{3}{7} + \frac{2}{7} \\ = 1\frac{5}{7}$$

$$b. \quad -\frac{1}{3} + \frac{2}{5} + \frac{3}{10} + \left(-\frac{1}{6}\right) = -\frac{1}{3} + \left(-\frac{1}{6}\right) + \frac{2}{5} + \frac{3}{10} \\ = -\frac{2}{6} + \left(-\frac{1}{6}\right) + \frac{4}{10} + \frac{3}{10} \\ = -\frac{3}{6} + \frac{7}{10} \\ = -\frac{1}{2} + \frac{7}{10} \\ = -\frac{5}{10} + \frac{7}{10} \\ = \frac{2}{10} \\ = \frac{1}{5}$$

$$31. \quad a. \quad \frac{2}{3} \times \frac{4}{5} \times \frac{5}{7} \times \frac{1}{8} = \frac{\cancel{2}^1 \times \cancel{4}^1 \times \cancel{5}^1 \times 1}{3 \times \cancel{5}_1 \times 7 \times \cancel{8}_1} \\ = \frac{1}{21}$$

$$b. \quad -1\frac{1}{8} \times 3\frac{3}{4} \times 1\frac{1}{3} \times \left(-1\frac{3}{5}\right) = -\frac{9}{8} \times \frac{15}{4} \times \frac{4}{3} \times \left(-\frac{8}{5}\right) \\ = + \left(\frac{\cancel{9}^1 \times \cancel{15}^1 \times \cancel{4}^1 \times \cancel{8}^1}{\cancel{8}_1 \times 4 \times \cancel{3}_1 \times \cancel{5}_1} \right) \\ = 9$$

$$32. \quad a. \quad (+4) + (-5) \times (-2) = (+4) + (10) \\ = 14$$

$$b. \quad \frac{3}{5} + \left[\frac{3}{10} - \left(-\frac{3}{4}\right) \right] = \frac{3}{5} + \left(\frac{3}{10} + \frac{3}{4} \right) \\ = \frac{3}{5} + \left(\frac{6}{20} + \frac{15}{20} \right) \\ = \frac{3}{5} + \left(\frac{21}{20} \right) \\ = \frac{\cancel{3}^1}{\cancel{5}_1} \times \frac{\cancel{20}^4}{\cancel{21}_7} \\ = \frac{4}{7}$$

$$\begin{aligned}
 \text{c. } -\frac{1}{2} + \frac{3}{4} + \left(-\frac{1}{4}\right) &= -\frac{1}{2} + \frac{3}{4} \times (-4) \\
 &= -\frac{1}{2} + (-3) \\
 &= -3\frac{1}{2}
 \end{aligned}$$

Note: It is recommended that students review the notes and do a few sample questions from the Practice Activities in the sections which correspond to the questions with which they experienced success.

It is recommended that students carefully study the notes and do most of the questions in the Practice Activities in the sections which correspond to the questions with which they experienced difficulties.

Questions	Skills	Sections
1, 2	interpreting integers	2
3, 4	comparing and ordering integers	2
5, 9.c.	adding integers	3
6, 9.a., 9.b.	subtracting integers	4
7, 9.d.	multiplying integers	5
8, 9.e.	dividing integers	6
10	interpreting fractions	7
11.a.-d.	writing equivalent fractions	7
11.e.-f.	expressing fractions as decimal numbers and vice versa	8
12	comparing fractions	8
13, 14, 15, 23.a., 23.d.,	adding and subtracting fractions and mixed numbers	9
16, 17, 18, 23.c.	multiplying fractions and mixed numbers	10
19, 20, 23.b.	dividing fractions and mixed numbers	11
21, 22	interpreting reciprocals	11
24	interpreting rational numbers	12
25	comparing rational numbers	12
26, 27	adding and subtracting rational numbers	13
28, 29	multiplying and dividing rational numbers	14
30, 31	using shortcuts to find sums and products	15
32	using order of operations	15

Assignment Answer Key**Part 1: Short-Answer Questions**

1. a. $\frac{1}{5}$ b. $\frac{9}{25}$ c. $\frac{11}{40}$ d. $\frac{3}{1000}$
2. a. 0.6 b. 0.875 c. 0.03 d. $0.\dot{1}$ or $0.\overline{1}$
3. a. $\frac{1}{5}$ b. $1\frac{3}{7}$ c. $\frac{4}{13}$ d. $\frac{2}{5}$
4. a. 8 b. -10 c. -12 d. $1\frac{1}{6}$
e. $-4\frac{7}{12}$ f. $-1\frac{5}{12}$ g. -0.3 h. -9.9
5. a. -4 b. 5 c. $1\frac{2}{5}$ d. $-1\frac{2}{7}$ e. $-4\frac{3}{4}$ f. -1.5 g. 13.57
6. a. 45 b. -84 c. $-\frac{1}{2}$ d. $\frac{9}{28}$ e. -10 f. $5\frac{3}{10}$ g. -0.189
7. a. -11 b. 12 c. $-1\frac{1}{8}$ d. $4\frac{2}{3}$ e. -8 f. 0.43 g. -1803
8. a. 18 b. 0.2 c. $\frac{7}{10}$ d. $14\frac{2}{5}$
9. a. $\frac{3}{5}$ b. $-\frac{3}{7}$ c. -8
10. a. $8 = \frac{+8}{+1}$ b. $0.6 = \frac{+6}{+10}$ c. $-2 = -\frac{2}{1}$ d. $3\frac{1}{4} = \frac{13}{4}$
 $= \frac{+13}{+4}$

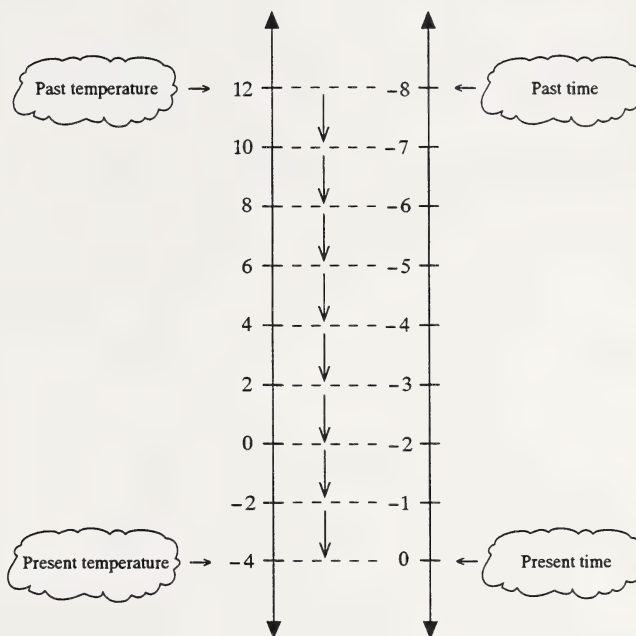
Part 2: Problems

1. $(8 \div 4 + 2) \div 2 + (9 \div 3 + 3) \div 3 - 4 = 0$

2. $12 - (-4) = 16$

$16 \div (-2) = -8$

It was 12°C eight hours ago.



3. $-13 + 10 - 8 = -11$

The resulting temperature was -11°C .

$$\begin{aligned}
 4 \div \frac{3}{4} &= \frac{16}{4} \div \frac{3}{4} & \text{or} & & 4 \div \frac{3}{4} &= 4 \times \frac{4}{3} \\
 &= \frac{16}{3} & & & &= \frac{16}{3} \\
 &= 5\frac{1}{3} & & & &= 5\frac{1}{3}
 \end{aligned}$$

Nathan can give $5\frac{1}{3}$ lessons in a 4-h shift.

5. **Method 1**

$$\frac{1}{6} \times 324 = 54$$

$$\frac{1}{4} \times 324 = 81$$

$$54 + 81 = 135$$

$$324 - 135 = 189$$

There are 189 students in grade 9.

Method 2

$$\begin{aligned}
 \frac{1}{6} + \frac{1}{4} &= \frac{2}{12} + \frac{3}{12} \\
 &= \frac{5}{12}
 \end{aligned}$$

$$\frac{12}{12} - \frac{5}{12} = \frac{7}{12}$$

$$\frac{7}{12} \times 324 = 189$$

There are 189 students in grade 9.

6. $3700 - 3417 = 283$

The altitude of the bottom of the mineshaft is 283 m above sea level.

$$-26 + 315 = 289$$

The altitude of the top of the radio tower is 289 m above sea level.

The top of the radio tower is at the greater altitude.

7. **Method 1**

$$\frac{1}{4} + \frac{1}{3} + \frac{1}{6} = \frac{3}{12} + \frac{4}{12} + \frac{2}{12}$$

$$= \frac{9}{12}$$

$$\frac{12}{12} - \frac{9}{12} = \frac{3}{12}$$

$$\frac{3}{12} \times \blacksquare = 42$$

$$\frac{3}{12} \times \boxed{168} = 42$$

The zookeeper cares for 168 animals altogether.

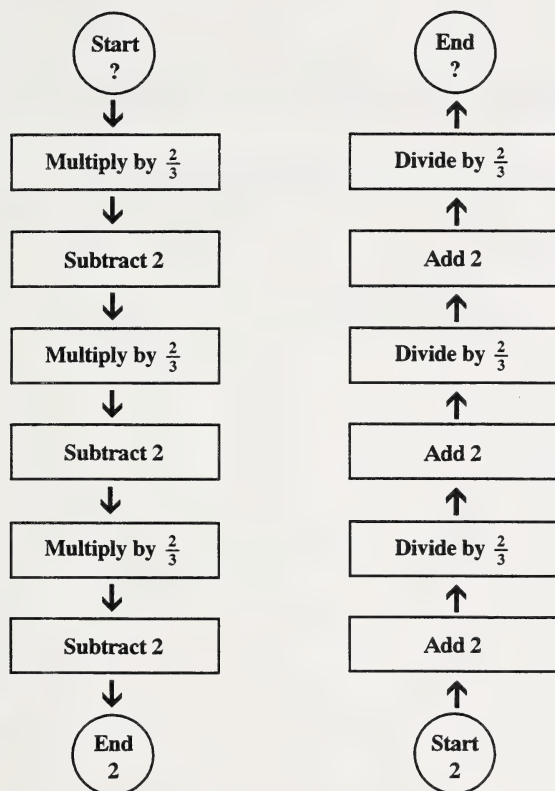
Method 2

The number has to be divisible by 4, 3, and 6. The number of animals is a multiple of 12.

	Number of Animals	Bears	Apes	Lions	Zebras	Test
Guess 1	72	18	24	12	42	$18 + 24 + 12 + 42 \neq 72$
Guess 2	108	27	36	18	42	$27 + 36 + 18 + 42 \neq 108$
Guess 3	144	36	48	24	42	$36 + 48 + 24 + 42 \neq 144$
Guess 4	168	42	56	28	42	$42 + 56 + 28 + 42 = 168$

The zookeeper cares for 168 animals altogether.

8.



Suzanne had 21 candies in the beginning.

Module 3: Powers

Gathering Materials

In this module the student will need the following items available from the Learning Resources Distributing Centre.

- Student Module 3: Powers
- Assignment Booklet for Module 3
- optional course audiocassette for Module 3
- required videocassette for Module 3 (*MATH MOVES: Number Theory*, ACCESS Network)

Students may use the following computer program if they have access to it and an Apple™ or compatible computer.

- *Computer Drill and Instruction: Mathematics, Level D* (SRA)

In addition, the students will need a binder and loose-leaf paper for their responses, a scientific calculator with a a^b/c key, and a geometry set.

Hold the Assignment Booklet for safekeeping until the student is ready for it.

Guiding the Student

Each section of Module 3 deals with a different aspect of powers. Sections 2 to 5 review previously developed skills. Section 11 is not required, but is included for enrichment. The Pretest in Section 1 will help you discover the student's strengths and weaknesses and make a personal study plan.

Ensure that the student reads the welcome pages and the module introduction, and does Section 1. Then check the answers to the Pretest (suggested answers follow). Help the student develop a personal study plan and schedule. An average student spends about six weeks of a 40-week school year on Module 3.

Use the guidelines provided at the beginning of this guide to monitor the student's progress and provide encouragement.

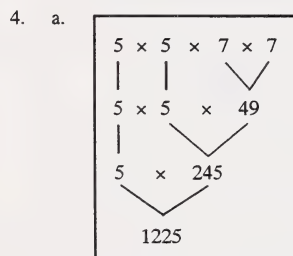
Refer to the suggested answers in the Appendix of Student Module 3: Powers when guiding the student through the Practice Activities in Module 3.

Section 1: Pretest

1. The factors of 48 are 1, 2, 3, 4, 6, 8, 12, 16, 24, and 48.
The factors of 56 are 1, 2, 4, 7, 8, 14, 28, and 56.
The factors of 80 are 1, 2, 4, 5, 8, 10, 16, 20, 40, and 80.

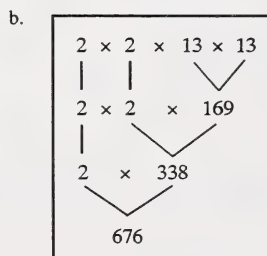
2. a. $48 = 2 \times 2 \times 2 \times 2 \times 3$ b. $56 = 2 \times 2 \times 2 \times 7$ c. $80 = 2 \times 2 \times 2 \times 2 \times 5$

3. a. Yes; 12 360 is an even number.
b. Yes; $1 + 2 + 3 + 6 + 0 = 12$ and 12 is divisible by 3.
c. Yes; 60 is divisible by 4 and the number is even.
d. Yes; the number ends in 0.
e. Yes; the number is even and it is divisible by 3.
f. Yes; 360 is divisible by 8 and the number is even.
g. No; $1 + 2 + 3 + 6 + 0 = 12$ and 12 is not divisible by 9.
h. Yes; the number ends in 0.



$$1225 = 5 \times 5 \times 7 \times 7$$

$$\begin{aligned}\sqrt{1225} &= 5 \times 7 \\ &= 35\end{aligned}$$



$$676 = 2 \times 2 \times 13 \times 13$$

$$\begin{aligned}\sqrt{676} &= 2 \times 13 \\ &= 26\end{aligned}$$

5. a. $\sqrt{3600} = 60$
 $\sqrt{3136} = ?$
 $\sqrt{2500} = 50$
 $\sqrt{3136}$ is between 50 and 60.

Key Press	Display
<div>3</div> <div>1</div> <div>3</div> <div>6</div> <div>$\sqrt{}$</div>	<div>56</div>

So, $\sqrt{3136} = 56$.

b. $\sqrt{1600} = 40$

$\sqrt{1024} = ?$

$\sqrt{900} = 30$

$\sqrt{1024}$ is between 30 and 40.

Key Press	Display
1 0 2 4 $\sqrt{}$	32

So, $\sqrt{1024} = 32$.

6. a. $\sqrt{900} = 30$

$\sqrt{537} = ?$

$\sqrt{400} = 20$

$\sqrt{537}$ is between 20 and 30.

Key Press	Display
5 3 7 $\sqrt{}$	23.17326045

So, $\sqrt{537} \approx 23.2$.

b. $\sqrt{1600} = 40$

$\sqrt{1284} = ?$

$\sqrt{900} = 30$

$\sqrt{1284}$ is between 30 and 40.

Key Press	Display
1 2 8 4 $\sqrt{}$	35.83294573

So, $\sqrt{1284} \approx 35.8$.

7. a. $23^2 = 23 \times 23$
= 529

You can use the square key on a calculator.

Key Press	Display
2 3 x^2	529

You can also use the automatic constant on a calculator.

b. $9^5 = 9 \times 9 \times 9 \times 9 \times 9$
= 59 049

You can use the power key on a calculator.

Key Press	Display
9 y^x 5 =	59049

You can also use the automatic constant on a calculator.

8. a.
$$\begin{array}{r} 3 \overline{)729} \\ 3 \overline{)243} \\ 3 \overline{)81} \\ 3 \overline{)27} \\ 3 \overline{)9} \\ 3 \overline{)3} \end{array}$$

So, $3^6 = 729$.

You can also use the automatic constant on a calculator to find the missing exponent.

b.
$$\begin{array}{r} 15 \overline{)11\,390\,625} \\ 15 \overline{)759\,375} \\ 15 \overline{)50\,625} \\ 15 \overline{)3375} \\ 15 \overline{)225} \\ 15 \overline{)15} \end{array}$$

So, $15^6 = 11\,390\,625$.

You can also use the automatic constant on a calculator to find the missing exponent.

$$\begin{aligned} 9. \quad a. \quad \left(\frac{3}{4}\right)^5 &= \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \\ &= \frac{243}{1024} \end{aligned}$$

$$\begin{aligned} b. \quad (1.5)^4 &= 1.5 \times 1.5 \times 1.5 \times 1.5 \\ &= 5.0625 \end{aligned}$$

$$\begin{aligned} c. \quad (-5)^3 &= (-5) \times (-5) \times (-5) \\ &= -125 \end{aligned}$$

$$\begin{aligned} d. \quad \left(-\frac{1}{2}\right)^4 &= \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \\ &= \frac{1}{16} \end{aligned}$$

$$\begin{aligned} e. \quad -2^5 &= (-2) \times (-2) \times (-2) \times (-2) \times (-2) \\ &= -32 \end{aligned}$$

$$\begin{aligned} f. \quad -3^6 &= (-3) \times (-3) \times (-3) \times (-3) \times (-3) \times (-3) \\ &= 729 \end{aligned}$$

$$\begin{aligned} 10. \quad a. \quad 3^5 \times 3^8 &= 3^{5+8} \\ &= 3^{13} \end{aligned}$$

$$\begin{aligned} b. \quad (-2)^2 \times (-2)^3 &= (-2)^{2+3} \\ &= (-2)^5 \end{aligned}$$

$$\begin{aligned} c. \quad \left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^5 &= \left(\frac{1}{2}\right)^{3+5} \\ &= \left(\frac{1}{2}\right)^8 \end{aligned}$$

$$\begin{aligned} 11. \quad a. \quad (5^4)^2 &= 5^{2 \times 4} \\ &= 5^8 \end{aligned}$$

$$\begin{aligned} b. \quad [(-2)^3]^5 &= (-2)^{5 \times 3} \\ &= (-2)^{15} \end{aligned}$$

$$\begin{aligned} c. \quad \left[\left(\frac{1}{2}\right)^3\right]^2 &= \left(\frac{1}{2}\right)^{2 \times 3} \\ &= \frac{1}{2}^6 \end{aligned}$$

$$\begin{aligned} 12. \quad a. \quad 8^6 \div 8^4 &= 8^{6-4} \\ &= 8^2 \end{aligned}$$

$$\begin{aligned} b. \quad (-2)^4 \div (-2) &= (-2)^{4-1} \\ &= (-2)^3 \end{aligned}$$

$$\begin{aligned} c. \quad \left(-\frac{3}{4}\right)^8 \div \left(-\frac{3}{4}\right)^2 &= \left(-\frac{3}{4}\right)^{8-2} \\ &= \left(-\frac{3}{4}\right)^6 \end{aligned}$$

13. In each case the powers do not have the same base.

$$\begin{aligned} 14. \quad a. \quad 3^8 &= (3^2)^4 \\ &= 9^4 \end{aligned}$$

$$\begin{aligned} b. \quad 8^6 &= (2^3)^6 \\ &= 2^{18} \\ &= (2^2)^9 \\ &= 4^9 \end{aligned}$$

$$\begin{aligned} 15. \quad 25^9 &= (5^2)^9 \\ &= 5^{18} \end{aligned}$$

So, $5^{20} > 25^9$.

16. a. $5^3 - 5^2 \times 2 = 125 - 25 \times 2$
 $= 125 - 50$
 $= 75$

b. $3 \times 9 + 9^2 = 3 \times 9 + 81$
 $= 27 + 81$
 $= 108$

c. $(2+3)^2 + 2+3 = 5^2 + 2+3$
 $= 25 + 2+3$
 $= 30$

d. $\frac{(-3)^2 + 6}{3} = \frac{9+6}{3}$
 $= \frac{15}{3}$
 $= 5$

e. $2 + \sqrt{49} - 9 = 2 + 7 - 9$
 $= 9 - 9$
 $= 0$

f. $\sqrt{3^2 + 4^2} = \sqrt{9+16}$
 $= \sqrt{25}$
 $= 5$

17. a. You can see that $10^0 = 1$ by using patterns.

Power Form	Standard Form	Pattern
10^4	10 000	$\left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \div 10$
10^3	1000	
10^2	100	
10^1	10	
10^0	1	$\div 10$

b. You can see that $10^3 = 1000$ and $10^{-3} = \frac{1}{1000}$ by using patterns.

Power Form	Standard Form	Pattern
10^4	10 000	$\left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \div 10$
10^3	1000	
10^2	100	
10^1	10	
10^0	1	$\div 10$
10^{-1}	$\frac{1}{10}$	$\div 10$
10^{-2}	$\frac{1}{100}$	$\div 10$
10^{-3}	$\frac{1}{1000}$	$\div 10$

1000 and $\frac{1}{1000}$ are reciprocals, so 10^3 and 10^{-3} are reciprocals.

18. a. $\overbrace{1000}^{3 \text{ zeros}} \times \overbrace{10\,000}^{4 \text{ zeros}} = \overbrace{10\,000\,000}^{7 \text{ zeros}}$

b. $0.001 \times 0.1 = 0.0001$

c. $\overbrace{1\,000\,000}^{6 \text{ zeros}} \div \overbrace{100}^{2 \text{ zeros}} = \overbrace{10\,000}^{4 \text{ zeros}}$

d. $0.000\,001 \div 0.01 = 0.0001$

19. a. $10^5 \times 10^8 = 10^{5+8}$
 $= 10^{13}$

b. $10^{-3} \times 10^2 = 10^{-3+2}$
 $= 10^{-1}$

c. $10^{-4} \times 10^{-5} = 10^{-4+(-5)}$
 $= 10^{-9}$

d. $10^8 \div 10^2 = 10^{8-2}$
 $= 10^6$

e. $10^5 \div 10^9 = 10^{5-9}$
 $= 10^{-4}$

f. $10^{-1} \div 10^{-3} = 10^{-1-(-3)}$
 $= 10^{-1+3}$
 $= 10^2$

20. a. $21\,300\,000 = 2.13 \times 10^7$ b. $4\,750\,000\,000 = 4.75 \times 10^9$
 c. $0.000\,000\,31 = 3.1 \times 10^{-7}$ d. $0.000\,000\,000\,526 = 5.26 \times 10^{-10}$
21. a. $3.14 \times 10^5 = 314\,000$ b. $2.12 \times 10^8 = 212\,000\,000$
 c. $3.08 \times 10^{-6} = 0.000\,003\,08$ d. $5.75 \times 10^{-7} = 0.000\,000\,575$
22. a. $12\,300\,000 \times 58\,000 = (1.23 \times 10^7) \times (5.8 \times 10^4)$
 $= 1.23 \times 5.8 \times 10^7 \times 10^4$
 $= 7.134 \times 10^{11}$
- b. $(1.2 \times 10^9) \times (7.3 \times 10^4) = 1.2 \times 7.3 \times 10^9 \times 10^4$
 $= 8.76 \times 10^{13}$
- c. $0.000\,006 \div 120\,000 = (6 \times 10^{-6}) \div (1.2 \times 10^5)$
 $= \frac{6 \times 10^{-6}}{1.2 \times 10^5}$
 $= \frac{6}{1.2} \times \frac{10^{-6}}{10^5}$
 $= 5 \times 10^{-6-5}$
 $= 5 \times 10^{-11}$
- d. $(5.34 \times 10^6) \div (8.9 \times 10^5) = \frac{5.34 \times 10^6}{8.9 \times 10^5}$
 $= \frac{5.34}{8.9} \times \frac{10^6}{10^5}$
 $= 0.6 \times 10^1$
 $= 6 \times 10^{-1} \times 10^1$
 $= 6 \times 10^0$
 $= 6 \times 1$
 $= 6$

Note: It is recommended that students review the notes and do a few sample questions from the Practice Activities in the sections which correspond to the questions with which they experienced success.

It is recommended that students carefully study the notes and do most of the questions in the Practice Activities in the sections which correspond to the questions with which they experienced difficulties.

Questions	Skills	Sections
1	factoring whole numbers	2
2	writing numbers as a product of prime factors	2
3	testing for divisibility	2
4, 5	calculating the square roots of perfect squares	3
6	approximating the square roots of non-perfect squares	4
7	evaluating powers	5
8	finding the missing exponent	5
9	evaluating more powers	6
10	writing a product of powers as a single power	7

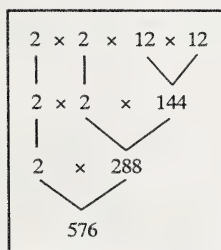
Questions	Skills	Sections
11	writing a power of a power as a single power	7
12	writing a quotient of a power as a single power	8
13	interpreting the property of powers	7, 8
14, 15	writing a power with a different base	7
16	using the rules for order of operations	8
17	interpreting powers of 10 with zero and negative exponents	9
18	calculating the products of very large and very small numbers	9
19	writing the products and quotients of powers of 10 as single powers	9
20, 21	writing numbers in scientific notation and vice versa	10
22	calculating the products and quotients of very large and very small numbers expressed in scientific notation	11

Assignment Answer Key

Part 1: Short-Answer Questions

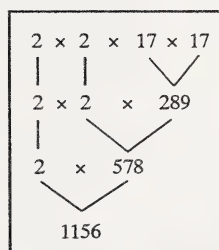
- 1, 2, 4, 8, 16, 32, 64
- Yes; the number is even.
 - Yes; $3 + 1 + 8 + 0 = 12$ and 12 is divisible by 3.
 - Yes; the number ends in 0.
 - Yes; the number is even and is divisible by 3.
 - No; the number is even, but the sum of the last three digits is not divisible by 8.
 - No; the sum of the digits is not divisible by 9.

3. a.



$$\begin{aligned}
 \sqrt{576} &= \sqrt{2 \times 12 \times 2 \times 12} \\
 &= \sqrt{24 \times 24} \\
 &= 24
 \end{aligned}$$

b.



$$\begin{aligned}
 1156 &= \sqrt{2 \times 17 \times 2 \times 17} \\
 &= \sqrt{34 \times 34} \\
 &= 34
 \end{aligned}$$

4. a. $\sqrt{400} = 20$

$\sqrt{365} = ?$

$\sqrt{100} = 10$

$\sqrt{365}$ is between 10 and 20.

Key Press	Display
$\boxed{3} \boxed{6} \boxed{5} \boxed{\sqrt{}}$	19.10497317

So, $\sqrt{365} \approx 19.1$.

b. $\sqrt{3600} = 60$

$\sqrt{3365} = ?$

$\sqrt{2500} = 50$

$\sqrt{3365}$ is between 50 and 60.

Key Press	Display
$\boxed{3} \boxed{3} \boxed{6} \boxed{5} \boxed{\sqrt{}}$	58.00862005

So, $\sqrt{3365} \approx 58.0$.

5. a. $21^2 = 21 \times 21$
 $= 441$

b. $\left(\frac{7}{8}\right)^3 = \frac{7}{8} \times \frac{7}{8} \times \frac{7}{8}$
 $= \frac{343}{512}$

c. $\left(-\frac{1}{2}\right)^4 = \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right)$
 $= \frac{1}{16}$

d. $(-1.7)^3 = (-1.7) \times (-1.7) \times (-1.7)$
 $= -4.913$

e. $-5^3 = -5 \times 5 \times 5$
 $= 125$

f. $-3^8 = -3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$
 $= -6561$

6. a.
$$\begin{array}{r} 9 \overline{)59\,049} \\ 9 \overline{)6561} \\ 9 \overline{)729} \\ 9 \overline{)81} \\ 9 \overline{)9} \end{array}$$

$9^5 = 59\,049$

b.
$$\begin{array}{r} -5 \overline{)15\,625} \\ -5 \overline{)3125} \\ -5 \overline{)625} \\ -5 \overline{)125} \\ -5 \overline{)25} \\ -5 \overline{)5} \end{array}$$

$(-5)^6 = 15\,625$

7. a. $6^5 \times 6^3 = 6^{5+3}$
 $= 6^8$

b. $(-3)^4 \times (-3)^5 = (-3)^{4+5}$
 $= (-3)^9$

c. $4^5 \times 2^3 = (2^2)^5 \times 2^3$ or Students may indicate that powers with different bases cannot be multiplied.
 $= 2^{10} \times 2^3$
 $= 2^{10+3}$
 $= 2^{13}$

d. $8^9 \div 8^4 = 8^{9-4}$
 $= 8^5$

e. $\left(-\frac{3}{4}\right)^7 \div \left(-\frac{3}{4}\right)^2 = \left(-\frac{3}{4}\right)^5$

f. $(3^2)^5 = 3^{5 \times 2}$
 $= 3^{10}$

g. $\left[(-2)^3\right]^4 = (-2)^{4 \times 3}$
 $= (-2)^{12}$

$$\begin{aligned}
 8. \quad 36^3 &= (6^2)^3 \\
 &= 6^{3 \times 2} \\
 &= 6^6
 \end{aligned}$$

So, 36^3 is greater than 6^5 .

$$\begin{aligned}
 9. \quad a. \quad 7^3 - 7^2 \times 2 &= 343 - 49 \times 2 \\
 &= 343 - 98 \\
 &= 245
 \end{aligned}$$

$$\begin{aligned}
 b. \quad (5-1)^2 - 3 &= 4^2 - 3 \\
 &= 16 - 3 \\
 &= 13
 \end{aligned}$$

$$\begin{aligned}
 c. \quad \left(\frac{4^2 + 2}{2}\right)^2 &= \left(\frac{16 + 2}{2}\right)^2 \\
 &= \left(\frac{18}{2}\right)^2 \\
 &= 9^2 \\
 &= 81
 \end{aligned}$$

10.

Power Form	Standard Form	Pattern
10^3	1000	$\left. \begin{array}{l} \\ \\ \\ \\ \\ \end{array} \right\} \begin{array}{l} +10 \\ +10 \\ +10 \\ +10 \\ +10 \end{array}$
10^2	100	
10^1	10	
10^0	1	
10^{-1}	$\frac{1}{10}$ or 0.1	
10^{-2}	$\frac{1}{100}$ or 0.01	

a. Using patterns, it is clear that $10^0 = 1$.

b. Using patterns, it is clear that $10^2 = 100$ and $10^{-2} = \frac{1}{100}$.

$$100 \times \frac{1}{100} = 1$$

$$\begin{aligned}
 10^2 \times 10^{-2} &= 10^{2+(-2)} \\
 &= 10^0 \\
 &= 1
 \end{aligned}$$

11. a. $248\,000\,000 = 2.48 \times 10^8$

b. $15\,000\,000 = 1.5 \times 10^7$

c. $0.000\,000\,099\,2 = 9.92 \times 10^{-8}$

d. $0.000\,000\,015 = 1.5 \times 10^{-8}$

12. a. $1.8 \times 10^8 = 180\,000\,000$
 b. $3.5 \times 10^7 = 35\,000\,000$
 c. $1.4 \times 10^{-9} = 0.000\,000\,001\,4$
 d. $2.7 \times 10^{-12} = 0.000\,000\,000\,002\,7$
13. This means that $350\,000 \times 124\,000 = 43\,400\,000\,000$.

Part 2: Problems

1. Students may make a diagram and use patterns.

Pattern

1	} $\times 3$ } $\times 3$ } $\times 3$ or } $\times 3$ } $\times 3$	3^0
3		3^1
9		3^2
27		3^3
81		3^4
243		3^5

$$3^5 = 243$$

The first floor has 243 apartments.

2. a. Students may use patterns.

Time	Bacteria
midnight	1000
1 a.m.	2000
2 a.m.	4000
3 a.m.	8000
4 a.m.	16 000

Pattern

} $\times 2$
} $\times 2$
} $\times 2$
} $\times 2$

or

Time	Bacteria
midnight	10000
1 a.m.	2×1000
2 a.m.	$2^2 \times 1000$
3 a.m.	$2^3 \times 1000$
4 a.m.	$2^4 \times 1000$

$$2^4 \times 1000 = 16 \times 1000 \\ = 16\,000$$

There will be 16 000 bacteria at 4 a.m.

- b. Students may use patterns.

Time	Bacteria	Pattern
midnight	1000	$\left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \div 2$
11 p.m.	500	
10 p.m.	250	
9 p.m.	125	

There were 125 bacteria at 9 p.m.

3. a. In the Student Module Booklet, students discovered that numbers that have two factors are prime numbers.

99 is not a prime number.

98 is not a prime number.

97 is a prime number.

The greatest two-digit number with exactly two factors is 97.

- b. In the Student Module Booklet, students discovered that numbers with exactly three factors are perfect squares.

$$32^2 = 1024 \quad (\text{This has more than three digits.})$$

$$31^2 = 961 \quad (\text{This has three digits.})$$

So, 961 is a perfect square.

4. In the Student Module Booklet, students discovered that all powers of whole numbers with an exponent of 5 end with the same digit that the base does.

Example

$$1^5 = 1$$

$$2^5 = 32$$

$$3^5 = 243$$

So, 11^5 ends in 1, 12^5 ends in 2, 13^5 ends in 3, and 14^5 ends in 4.

$$1 + 2 + 3 + 4 = 10$$

So, $11^5 + 12^5 + 13^5 + 14^5$ ends in 0.

$$\begin{aligned} 5. \quad a. \quad 12 \times 12 &= 12^2 \\ &= 144 \end{aligned}$$

A gross is 144 items.

$$\begin{aligned} b. \quad 12 \times 12 \times 12 &= 12^3 \\ &= 1728 \end{aligned}$$

A great gross is 1728 items.

Year	Amount
1990	100.0
1991	100.0
1992	100.0
1993	100.0
1994	100.0

Year	Amount
1990	100.0
1991	100.0
1992	100.0
1993	100.0
1994	100.0

Year	Amount
1990	100.0
1991	100.0
1992	100.0
1993	100.0
1994	100.0

Year	Amount
1990	100.0
1991	100.0
1992	100.0
1993	100.0
1994	100.0



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